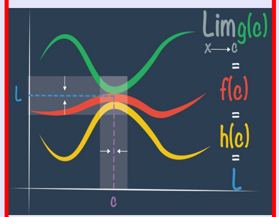
# Calculus I

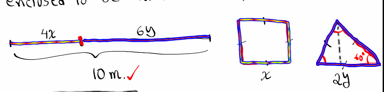
# Lecture 40




Feb 19-8:47 AM

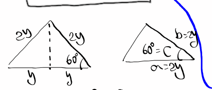
A piece of wire is 10m long. It is cut into two pieces. One piece is square and the other Piece is an equilateral triangle.

How should we cut it to have total area enclosed to be minimum and/or maximum?




$\boxed{4x + 6y = 10}$

Total Area = $A_{square} + A_{Triangle} = \boxed{x^2 + y^2\sqrt{3}}$




Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{\cancel{2}} \cdot \cancel{2}y \cdot 2y \cdot \sin 60^\circ$

$= \cancel{2} y^2 \cdot \frac{\sqrt{3}}{\cancel{2}} = \boxed{y^2\sqrt{3}}$

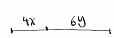
$6y = 10 - 4x$

$y = \frac{10-4x}{6} = \frac{5-2x}{3}$

$\boxed{f(x) = x^2 + \left(\frac{5-2x}{3}\right)^2 \cdot \sqrt{3}}$

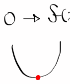
$\frac{(5-2x)^2 \cdot \sqrt{3}}{9}$

$f'(x) = 2x + \frac{\sqrt{3}}{9}\left(2 \cdot (5-2x)^1 \cdot (-2)\right)$




$\boxed{f'(x) = 2x - \frac{4\sqrt{3}}{9}(5-2x)}$

$f''(x) = 2 - \frac{4\sqrt{3}}{9}(-2) = 2 + \frac{8\sqrt{3}}{9} > 0 \rightarrow f(x)$ is C.U.



$f'(x) = 0$

$2x - \frac{4\sqrt{3}}{9}(5-2x) = 0$

$18x - 4\sqrt{3}(5-2x) = 0$

$18x - 20\sqrt{3} + 8\sqrt{3}\,x = 0$

$18x + 8\sqrt{3}\,x = 20\sqrt{3}$

$(18 + 8\sqrt{3})x = 20\sqrt{3}$

$x = \frac{20\sqrt{3}}{18 + 8\sqrt{3}}$

$\rightarrow x = \frac{\overset{10}{\cancel{20}}\sqrt{3}}{\cancel{2}(9+4\sqrt{3})}$ $\quad x = \frac{10\sqrt{3}}{9+4\sqrt{3}}$

Also find $f(0)$ & $f(10)$

Apr 23-9:32 AM

$x = \dfrac{10\sqrt{3}}{9+4\sqrt{3}}$ $\quad x \approx 1.1$ $\quad \boxed{f'(x) = 2x - \dfrac{4\sqrt{3}}{9}(5-2x)}$

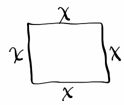
| $x$ | | 1.1 | |
|---|---|---|---|
| $f'(x)$ | $-$ | | $+$ |
| $f''(x)$ | $+$ | | $+$ |
| $f(x)$ | ↘ | | ↗ |

$4 - \dfrac{4\sqrt{3}}{9} > 0$

$-\dfrac{4\sqrt{3}}{9} \cdot 5 < 0$

min. at $x = \dfrac{10\sqrt{3}}{9+4\sqrt{3}}$

what if NO square $\to x = 0$

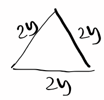



Area

$f(x) = x^2 + \dfrac{\sqrt{3}}{9}(5-2x)^2$

$f(0) = 0 + \dfrac{\sqrt{3}}{9}(5-2(0))^2 = \boxed{\dfrac{25\sqrt{3}}{9}} \approx \boxed{4.81}$

what if No triangle $\to y = 0$




$4x + 6y = 10$

$4x = 10$

$x = 2.5$

Area ✓

$f(2.5) = 2.5^2 + 0 = \boxed{6.25}$

Max. Area

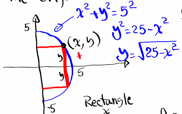
find $f(x)$ where $f'(x) = 0$ at $x \approx 1.1$

$f(1.1) = 1.1^2 + \dfrac{\sqrt{3}}{9}(5-2(1.1))^2$

$\approx \boxed{2.72}$ ✓ Min. Area

Apr 25-9:01 AM

Consider the right have of a circle centered at the origin and radius 5in. Find the $\underbrace{\text{largest rectangle}}_{\text{Max. Area}}$ that can be inscribed in this Semicircle.

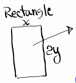



$x^2 + y^2 = 5^2$

$y^2 = 25 - x^2$

$y = \sqrt{25 - x^2}$

Rectangle




$A = x \cdot 2y = 2xy$

$A = 2x \cdot \sqrt{25 - x^2}$

$f(x) = 2x(25 - x^2)^{1/2}$ Needs to be maximized

$f'(x) = 2\left[1 \cdot (25 - x^2)^{1/2} + x \cdot \frac{1}{2}(25 - x^2)^{-1/2} \cdot (-2x)\right]$

$f'(x) = 2\left[\sqrt{25 - x^2} - \dfrac{x^2}{\sqrt{25 - x^2}}\right] = 2 \cdot \dfrac{25 - x^2 - x^2}{\sqrt{25 - x^2}}$

$f'(x) = \dfrac{2(25 - 2x^2)}{\sqrt{25 - x^2}}$

$f'(x) = 0 \to 25 - 2x^2 = 0 \quad x = \dfrac{5}{\sqrt{2}}$

$f'(x)$ und. $\to x = 5 \to$ No rectangle

$\dfrac{5}{\sqrt{2}} \approx 3.5$

| $x$ | | $\frac{5}{\sqrt{2}}$ | |
|---|---|---|---|
| $f'(x)$ | $+$ | | $-$ |
| $f(x)$ | ↗ | | ↘ |

Max. Point

take $x = 4$ $\quad f'(4) = -$

take $x = 1$ $\quad f'(1) = +$

Area $= 2xy = 2x\sqrt{25 - x^2}$

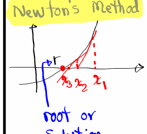
when $x = \dfrac{5}{\sqrt{2}}$ $\quad A = 2 \cdot \dfrac{5}{\sqrt{2}} \cdot \sqrt{25 - \left(\dfrac{5}{\sqrt{2}}\right)^2}$

$= 2 \cdot \dfrac{5}{\sqrt{2}} \cdot \sqrt{25 - \dfrac{25}{2}} = 2 \cdot \dfrac{5}{\sqrt{2}} \cdot \sqrt{\dfrac{25}{2}}$

$= \cancel{2} \cdot \dfrac{5}{\cancel{\sqrt{2}}} \cdot \dfrac{5}{\cancel{\sqrt{2}}} \Rightarrow$ Max. Area $\boxed{25 \text{ in}^2}$

Google

Newton's Method




$x_1$ First guess

$x_2$ like Second guess

$x_3$ " Third guess

Apr 25-9:12 AM

Class QZ 19

$f(x) = x^5 - 5x$

1) Find $f'(x)$, Solve $f'(x)=0$

$f'(x) = 5x^4 - 5$
$= 5(x^4 - 1)$
$= 5(x^2+1)(x+1)(x-1)$

$f'(x)=0 \rightarrow x = \pm 1$

2) Find $f''(x)$, Solve $f''(x)=0$

$f''(x) = 20x^3$
$f''(x)=0 \rightarrow x=0$

3) Complete the Sign chart

| $x$ | $-\infty$ | $-1$ | | $0$ | | $1$ | | $\infty$ |
|---|---|---|---|---|---|---|---|---|
| $f'(x)$ | + | • | − | | − | • | + | |
| $f''(x)$ | − | | − | • | + | | + | |
| $f(x)$ | ↗ | | ↘ | | ↘ | | ↗ | |

4) Discuss in interval notation

increasing $(-\infty,-1)\cup(1,\infty)$

Decreasing $(-1,1)$

Concave up $(0,\infty)$

Concave down $(-\infty,0)$